CE 342 - Fall 2002 - Exam #3
Problem 3 - Solution
RJN 12/3/2

Given $kip := 1000 \cdot lbf$

$w := 1.2 \cdot \frac{kip}{ft}$

$L_{ab} := 4 \cdot ft$

$L_{bc} := 24 \cdot ft$

From symmetry, we note that

$\theta_c := 0 \cdot rad$

and

$\theta_d = -\theta_b$ and $M_{dc} = -M_{bc}$

Therefore we will be able to model half the structure and determine the remaining reactions, etc. from symmetry.

Calculate the fixed end moments for span bc.

$FEM_{bc} := \frac{w \cdot L_{bc}^2}{12}$ $FEM_{cb} := -FEM_{bc}$ $FEM_{bc} = 57.6\,kip \cdot ft$

Calculate the moment due to the overhang

$M_{ba} := -\frac{w \cdot L_{ab}^2}{2}$ $M_{ba} = -9.6\,kip \cdot ft$

Write the slope-deflection equations for bc

$$M_{bc} = 2 \cdot \frac{E \cdot I}{L_{bc}} \cdot (2 \cdot \theta_b + \theta_c) + FEM_{bc}$$

$$M_{cb} = 2 \frac{E \cdot I}{L_{bc}} \cdot (2 \cdot \theta_c + \theta_b) + FEM_{cb}$$

Since θ_c = 0 rad, these become

$$M_{bc} = 2 \cdot \frac{E \cdot I}{L_{bc}} \cdot (2 \cdot \theta_b) + FEM_{bc}$$

$$M_{cb} = 2 \frac{E \cdot I}{L_{bc}} \cdot (\theta_b) + FEM_{cb}$$

Writing the equilibrium equation for joint a

$$M_{ba} + M_{bc} = 0$$

Substituting for M_{bc},

$$M_{ba} + 2 \cdot \frac{E \cdot I}{L_{bc}} \cdot (2 \cdot \theta_b) + FEM_{bc} = 0$$

Solving for θ_b,

$$\theta_b = \frac{-1}{4} \cdot \frac{(M_{ba} + FEM_{bc})}{E \cdot I} \cdot L_{bc}$$

Substituting this in the equations for M_{bc}

$$M_{bc} = 2 \cdot \frac{E \cdot I}{L_{bc}} \cdot \left[2 \cdot \left[\frac{-1}{4} \cdot \frac{(M_{ba} + FEM_{bc})}{E \cdot I} \cdot L_{bc} \right] \right] + FEM_{bc}$$

Simplifying

$M_{bc} := (-M_{ba} - FEM_{bc} + FEM_{bc})$ $\quad M_{bc} = 9.6\,kip \cdot ft$ Check

Substituting θ_b in the equations for M_{cb}

$$M_{cb} = 2\frac{E \cdot I}{L_{bc}} \cdot \left[\frac{-1}{4} \cdot \frac{(M_{ba} + FEM_{bc})}{E \cdot I} \cdot L_{bc} \right] + FEM_{cb}$$

Simplifying

$M_{cb} := \frac{-1}{2} \cdot M_{ba} - \frac{1}{2} \cdot FEM_{bc} + FEM_{cb}$ $\quad M_{cb} = -81.6\,kip \cdot ft$

Calculating shear forces

Taking a free-body diagram of ab and summing forces in the vertical direction

$S_{ba} := -w \cdot L_{ab}$ $\quad S_{ba} = -4.8\,kip$

Taking a free-body diagram of span bc and summing moments about point b.

$$-S_{cb} \cdot L_{bc} - w \cdot \frac{L_{bc}^2}{2} + M_{bc} + M_{cb} = 0$$

Solving for S_{cb}

$S_{cb} := \frac{1}{2} \cdot \frac{\left(-w \cdot L_{bc}^2 + 2 \cdot M_{bc} + 2 \cdot M_{cb}\right)}{L_{bc}}$ $\quad S_{cb} = -17.4\,kip$

Summing forces in the vertical direction

$$S_{bc} - w \cdot L_{bc} - S_{cb} = 0$$

Solving for S_{cb}

$S_{bc} := S_{cb} + w \cdot L_{bc}$ $\quad S_{bc} = 11.4\,kip$

Take a free-body diagram of joint b, and sum forces in the vertical direction. Note that the shears calculated above use deformation sign conventions which must be converted to coordinate based sign conventions.

$$S_{ba} - S_{bc} + b_y = 0$$

$b_y := -S_{ba} + S_{bc}$ $\quad b_y = 16.2\,kip$

Calculate the vertical reaction at c from a free body diagram of joint c.

$$c_y + 2 \cdot S_{cb} = 0$$

$$c_y := -2 \cdot S_{cb} \qquad c_y = 34.8\,\text{kip}$$

Calculate the point of zero shear in span bc using similar triangles.

$$\frac{x_0}{S_{bc}} = \frac{L_{bc}}{\left(S_{bc} - S_{cb}\right)}$$

$$x_0 := \frac{-L_{bc}}{\left(-S_{bc} + S_{cb}\right)} \cdot S_{bc} \qquad x_0 = 9.5\,\text{ft}$$

Calculate the moment at this point

$$M_{max} := -M_{bc} + \frac{1}{2} \cdot x_0 \cdot S_{bc} \qquad M_{max} = 44.55\,\text{kip} \cdot \text{ft}$$

Plotting the shear and moment diagrams