CE 342 - Fall 2002 - Exam #3
Problem 2 - Solution
RJN 12/3/2

Release the moment at d

Calculate the rotation at d due to the thermal gradients in the members.
Apply a unit couple at d, calculate the reactions and draw the moment diagrams.

Reactions

Moment diagrams for the virtual load

The virtual work equation for displacements (rotations) due to thermal gradients is

$$\theta_{d\Delta T}\cdot 1\cdot kip\cdot in = \int_a^b \frac{-\alpha\cdot\Delta T}{h}\cdot M_v\,dx + \int_d^c \frac{-\alpha\cdot\Delta T}{h}\cdot M_v\,dx$$

If the local coordinates for ab go from a to b, the temperature gradient is positive.

$\Delta T_{ab} := 150\cdot R - (-20\cdot R)$ Mathcad uses absolute temperature (Rankine scale) instead of Farenheit. Since we only care about change in temperature, this isn't a problem.

and M_v is positive.

If the local coordinates for dc go from d to c, the temperature gradient is negative.

$\Delta T_{dc} := -20\cdot R - 150\cdot R$

and M_v is negative.

Using the product integral table

$$\theta_{d\Delta T}\cdot 1\cdot kip\cdot in = \frac{L_{ab}}{2}\cdot 1\cdot kip\cdot in\cdot\frac{-\alpha\cdot\Delta T_{ab}}{h_{ab}} + \frac{L_{dc}}{2}\cdot(-1\cdot kip\cdot in)\cdot\frac{-\alpha\cdot\Delta T_{dc}}{h_{dc}}$$

The problem statement gives us

$\alpha := \frac{6.5\cdot 10^{-6}}{R}$ $\quad kip := 1000\cdot lbf$ $\quad h_{ab} := 12\cdot in$ $\quad h_{dc} := h_{ab}$ $\quad L_{ab} := 20\cdot ft$ $\quad L_{dc} := L_{ab}$

$E := 29000\cdot\frac{kip}{in^2}$ $\quad I_{ab} := 500\cdot in^4$ $\quad I_{dc} := I_{ab}$

$$\theta_{d\Delta T} := \left[\frac{L_{ab}}{2}\cdot 1\cdot kip\cdot in\cdot\frac{-\alpha\cdot\Delta T_{ab}}{h_{ab}} + \frac{L_{dc}}{2}\cdot(-1\cdot kip\cdot in)\cdot\frac{-\alpha\cdot\Delta T_{dc}}{h_{dc}}\right]\cdot\frac{1}{1\cdot kip\cdot in} \qquad \theta_{d\Delta T} = -0.022\ rad$$

Calculate the rotation due to the redundant moment M_d.

The moment diagrams for Md can be scaled from the moment diagram for the unit moment.

and the virtual work equation becomes

$$\theta_{dMd}\cdot 1\cdot kip\cdot in = \int_a^b \frac{M\cdot M_v}{E\cdot I}\,dx + \int_d^c \frac{M\cdot M_v}{E\cdot I}\,dx$$

Using product integrals

$$\theta_{dMd} = \frac{1}{1\cdot kip\cdot in}\cdot\left[\frac{L_{ab}}{3}\cdot\frac{M_d\cdot 1\cdot kip\cdot in}{E\cdot I_{ab}} + \frac{L_{dc}}{3}\cdot\frac{-M_d}{E\cdot I_{dc}}\cdot(-1\cdot kip\cdot in)\right]$$

The compatibility equation is

$$\theta_{d\Delta T} + \theta_{dMd} = 0$$

Substituting

$$\theta_{d\Delta T} + \left(\frac{L_{ab}}{3}\cdot\frac{M_d}{E\cdot I_{ab}} + \frac{L_{dc}}{3}\cdot\frac{M_d}{E\cdot I_{dc}}\right) = 0$$

$$M_d := -3\cdot\frac{\theta_{d\Delta T}}{L_{ab}\cdot I_{dc} + L_{dc}\cdot I_{ab}}\cdot E\cdot I_{ab}\cdot I_{dc} \qquad M_d = 166.901\ kip\cdot ft$$

Scaling the reactions from the unit load case:

$$d_x := \frac{M_d}{240\cdot in} \qquad d_x = 8.345\ kip$$

Alternate solution - release the horizontal reaction in the pin at c. Since bc is now essentially simply supported, we can show that there is no shear in bc, only axial forces. Calculate the relative displacement at c due to the thermal forces. Apply an internal horizontal force at c. The moment diagrams are triangular (similar to the previous solution) with a maximum moment of 240 kip inches at a and d. Therefore the thermal displacements are

$$\delta_{d\Delta T} := \left[\frac{L_{ab}}{2}\cdot 240\cdot kip\cdot in\cdot\frac{-\alpha\cdot\Delta T_{ab}}{h_{ab}} + \frac{L_{dc}}{2}\cdot(-240\cdot kip\cdot in)\cdot\frac{-\alpha\cdot\Delta T_{dc}}{h_{dc}}\right]\cdot\frac{1}{1\cdot kip} \qquad \delta_{d\Delta T} = -5.304\ in$$

The relative displacement due to the redundant force can be found from beam tables.

$$\delta_{cx} = 2\cdot\frac{c_x\cdot L_{ab}^{\ 3}}{3\cdot E\cdot I_{ab}}$$

Since the relative displacement is zero

$$\delta_{d\Delta T} + \delta_{cx} = 0$$

or

$$\delta_{d\Delta T} + 2\cdot\frac{c_x\cdot L_{ab}^{\ 3}}{3\cdot E\cdot I_{ab}} = 0$$

solving for c_x.

$$c_x := \frac{-3}{2}\cdot\frac{\delta_{d\Delta T}}{L_{ab}^{\ 3}}\cdot E\cdot I_{ab} \qquad c_x = 8.345\ kip$$

and

$$M_d := c_x\cdot L_{ab} \qquad M_d = 166.901\ kip\cdot ft$$

Risa stuff

$$M := \frac{\alpha\cdot\Delta T_{ab}\cdot E\cdot I_{ab}}{h_{ab}} \qquad M = 111.267\ kip\cdot ft$$

$$M + 55.6\cdot kip\cdot ft = 166.867\ kip\cdot ft$$