CE 342 - Fall 2002 - Exam #3
Problem 1 Solution
RJN 12/3/2

$psf := \frac{lbf}{ft^2}$

Given:

Floor spacing $s := 12 \cdot ft$

Wall dimension East side $b := 72 \cdot ft$

Windward pressure $p_w := 10 \cdot psf$

Leeward pressure $p_l := 5 \cdot psf$

Calculate the wind loads on the bracing at each floor level using tributary areas.

Roof level

Tributary area

$A_r := \frac{s}{2} \cdot \frac{b}{2}$

On the windward side

$F_{rw} := A_r \cdot p_w$ $F_{rw} = 2160\,lbf$

On the leeward side

$F_{rl} := A_r \cdot p_l$ $F_{rl} = 1080\,lbf$

All floor levels

Tributary area

$A_f := s \cdot \frac{b}{2}$

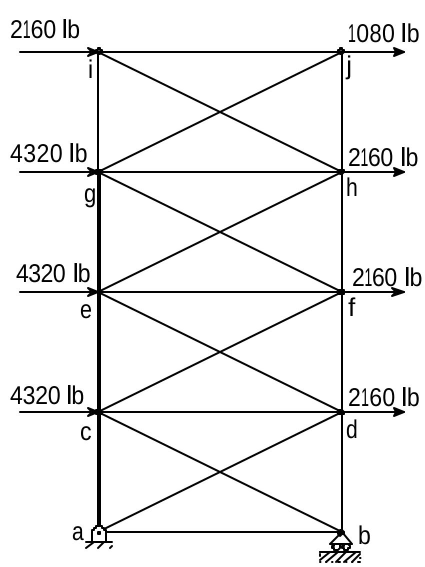
On the windward side

$F_{fw} := A_f \cdot p_w$ $F_{fw} = 4320\,lbf$

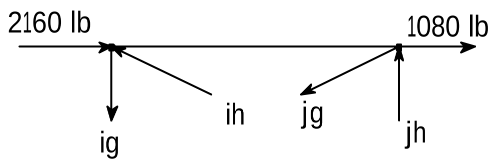
On the leeward side

$F_{fl} := A_f \cdot p_l$ $F_{fl} = 2160\,lbf$

Since the beams and columns are pinned, only the braced frame between lines B and C on frame line 4 carry these lateral loads. Therefore, the bracing loads are



Take a free-body diagram of the roof level.



Trigonometry for vector algebra

$$hyp := \sqrt{(20 \cdot ft)^2 + (12 \cdot ft)^2}$$

$$hyp = 23.324\,ft$$

For the approximate solution, assume that diagonal members ih and jg have equal stiffness, and therefore carry equal forces.

Sum horizontal forces

$$2160 \cdot lbf + 1080 \cdot lbf - \frac{20 \cdot ft}{hyp} \cdot ih \cdot 2 = 0$$

$ih := 81 \cdot \frac{lbf}{ft} \cdot hyp$ $\quad ih = 1889.228\,lbf$ compression

$jg := ih$ $\quad jg = 1889.228\,lbf$ tension

Sum moments about point i

$$jh \cdot 20 \cdot ft - jg \cdot \frac{12 \cdot ft}{hyp} \cdot 20 \cdot ft = 0$$

$jh := 12 \cdot jg \cdot \frac{ft}{hyp}$ $\quad jh = 972\,lbf$ compression

Sum vertical forces

$$jh - jg \cdot 12 \cdot \frac{ft}{hyp} + ih \cdot \frac{12 \cdot ft}{hyp} - ig = 0$$

$ig := \frac{-(-jh \cdot hyp + 12 \cdot jg \cdot ft - 12 \cdot ft \cdot ih)}{hyp}$ $\quad ig = 972\,lbf$ tension

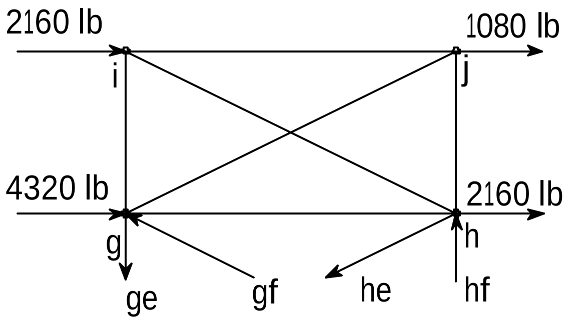
Take a free-body of joint i

Sum horizontal forces

$$2160 \cdot lbf - ih \cdot \frac{20 \cdot ft}{hyp} - ij = 0$$

$ij := 20 \cdot \frac{(108 \cdot lbf \cdot hyp - ft \cdot ih)}{hyp}$ $\quad ij = 540\,lbf$ compression

Take a free-body diagram of the roof and fourth floor



Sum horizontal forces

$$2160 \cdot lbf + 1080 \cdot lbf + 2160 \cdot lbf + 4320 \cdot lbf - \frac{20 \cdot ft}{hyp} \cdot gf \cdot 2 = 0$$

$gf := 243 \cdot \frac{lbf}{ft} \cdot hyp$ $\quad gf = 5667.685\,lbf$ compression

$he := gf$ $\quad he = 5667.685\,lbf$ tension

Sum moments about point g

$$hf \cdot 20 \cdot ft - he \cdot \frac{12 \cdot ft}{hyp} \cdot 20 \cdot ft - (2160 \cdot lbf + 1080 \cdot lbf) \cdot 12 \cdot ft = 0$$

$hf := 12 \cdot \frac{(he \cdot ft + 162 \cdot lbf \cdot hyp)}{hyp}$ $\quad hf = 4860\,lbf$ compression

Sum vertical forces

$$hf - he \cdot 12 \cdot \frac{ft}{hyp} + gf \cdot \frac{12 \cdot ft}{hyp} - ge = 0$$

$ge := \frac{(hf \cdot hyp - 12 \cdot he \cdot ft + 12 \cdot ft \cdot gf)}{hyp}$ $\quad ge = 4860\,lbf$ tension

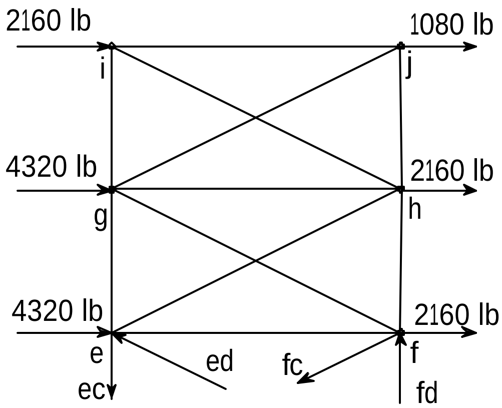
Take a free-body of joint g

Sum horizontal forces

$$4320 \cdot lbf - gf \cdot \frac{20 \cdot ft}{hyp} + jg \cdot \frac{20 \cdot ft}{hyp} - hg = 0$$

$hg := 20 \cdot \frac{(216 \cdot lbf \cdot hyp - ft \cdot gf + jg \cdot ft)}{hyp}$ $\quad hg = 1080\,lbf$ compression

Take a free-body diagram of the roof and third and fourth floors



Sum horizontal forces

$$2160 \cdot lbf + 1080 \cdot lbf + 2 \cdot 2160 \cdot lbf + 2 \cdot 4320 \cdot lbf - \frac{20 \cdot ft}{hyp} \cdot ed \cdot 2 = 0$$

$ed := 405 \cdot \frac{lbf}{ft} \cdot hyp$ $\quad ed = 9446.142\, lbf$ compression

$fc := ed$ $\quad fc = 9446.142\, lbf$ tension

Sum moments about point e

$$fd \cdot 20 \cdot ft - fc \cdot \frac{12 \cdot ft}{hyp} \cdot 20 \cdot ft - (2160 \cdot lbf + 1080 \cdot lbf) \cdot 24 \cdot ft - (4320 \cdot lbf + 2160 \cdot lbf) \cdot 12 \cdot ft = 0$$

$fd := 12 \cdot \frac{(fc \cdot ft + 648 \cdot lbf \cdot hyp)}{hyp}$ $\quad fd = 12636\, lbf$ compression

Sum vertical forces

$$fd - fc \cdot 12 \cdot \frac{ft}{hyp} + ed \cdot \frac{12 \cdot ft}{hyp} - ec = 0$$

$ec := \frac{-(-fd \cdot hyp + 12 \cdot fc \cdot ft - 12 \cdot ft \cdot ed)}{hyp}$ $\quad ec = 12636\, lbf$ tension

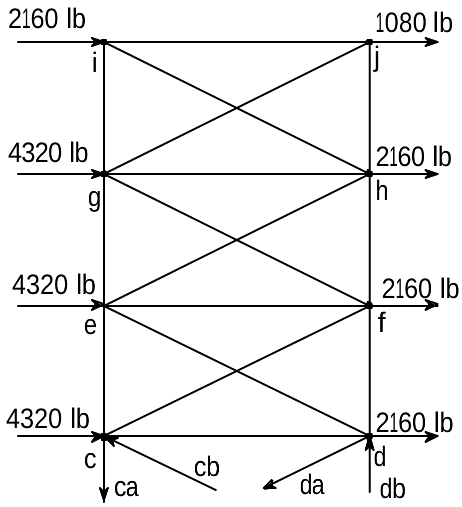
Take a free-body of joint e

Sum horizontal forces

$$4320 \cdot lbf - ed \cdot \frac{20 \cdot ft}{hyp} + he \cdot \frac{20 \cdot ft}{hyp} - ef = 0$$

$ef := 20 \cdot \frac{(216 \cdot lbf \cdot hyp - ft \cdot ed + he \cdot ft)}{hyp}$ $\quad ef = 1080\, lbf$ compression

Take a free-body diagram of the roof and the second through the fourth floors



Sum horizontal forces

$$2160\cdot lbf + 1080\cdot lbf + 3\cdot 2160\cdot lbf + 3\cdot 4320\cdot lbf - \frac{20\cdot ft}{hyp}\cdot cb\cdot 2 = 0$$

$cb := 567\cdot\frac{lbf}{ft}\cdot hyp$ $\quad cb = 13224.6\,lbf$ compression

$da := cb$ $\quad da = 13224.6\,lbf$ tension

Sum moments about point c

$$db\cdot 20\cdot ft - da\cdot\frac{12\cdot ft}{hyp}\cdot 20\cdot ft - (2160\cdot lbf + 1080\cdot lbf)\cdot 36\cdot ft - (4320\cdot lbf + 2160\cdot lbf)\cdot 24\cdot ft - (4320\cdot lbf + 2160\cdot lbf)\cdot 12\cdot ft = 0$$

$db := 12\cdot\frac{(da\cdot ft + 1458\cdot lbf\cdot hyp)}{hyp}$ $\quad db = 24300\,lbf$ compression

Sum vertical forces

$$db - da\cdot 12\cdot\frac{ft}{hyp} + cb\cdot\frac{12\cdot ft}{hyp} - ca = 0$$

$ca := \frac{(db\cdot hyp - 12\cdot da\cdot ft + 12\cdot ft\cdot cb)}{hyp}$ $\quad ca = 24300\,lbf$ tension

Take a free-body of joint c

Sum horizontal forces

$$4320\cdot lbf - cb\cdot\frac{20\cdot ft}{hyp} + fc\cdot\frac{20\cdot ft}{hyp} - cd = 0$$

$cd := 20\cdot\frac{(216\cdot lbf\cdot hyp - ft\cdot cb + fc\cdot ft)}{hyp}$ $\quad cd = 1080\,lbf$ compression