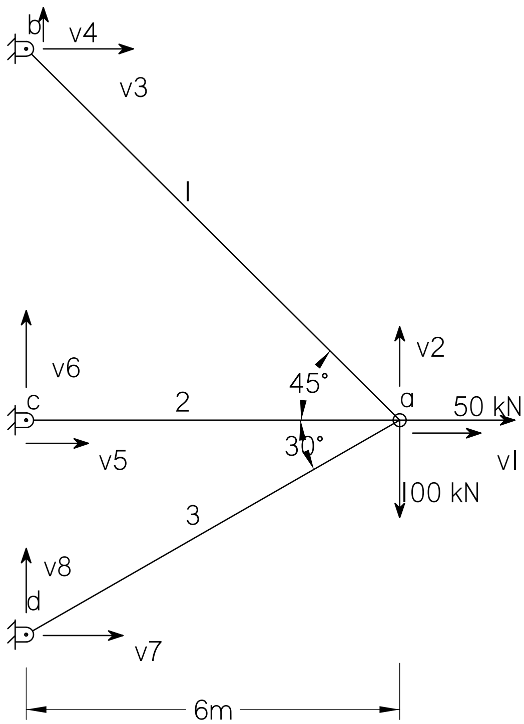
CE 342 - Truss problem

$C := K$ $kN := 1000 \cdot newton$ $GPa := 10^9 \cdot \frac{newton}{m^2}$

Find the nodal displacements, support reactions, and member forces for the structure shown below.




$ne := 3$ Number of elements

$i := 1 .. ne$

$DOFn := 2$ Number of degrees of freedom per node

$Nn := 4$ Number of nodes in system

$DOF := Nn \cdot DOFn$ Total number of DOF for system $DOF = 8$

## Element Properties:

$E_i := 200 \cdot GPa$ Modulus of Elasticity for all members, kN/mm2

Area of members, m2

$A_1 := 30 \cdot 10^{-4} \cdot m^2$

$A_2 := 20 \cdot 10^{-4} \cdot m^2$

$A_3 := 15 \cdot 10^{-4} \cdot m^2$

thermal coefficients

$\alpha_i := 1.2 \cdot \frac{10^{-5}}{C}$

$\Delta T := 45 \cdot C$

Nodal coordinates

$$x := \begin{pmatrix} 6 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot m \qquad y := \begin{pmatrix} 6\cdot\tan(30\cdot deg) \\ 6\cdot\tan(30\cdot deg) + 6\cdot\tan(45\cdot deg) \\ 6\cdot\tan(30\cdot deg) \\ 0 \end{pmatrix} \cdot m$$

Member connectivity

Specify i and j nodes for each member.

$$e_1 := \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad e_2 := \begin{pmatrix} 1 \\ 3 \end{pmatrix} \qquad e_3 := \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

Find direction cosines

X vector is from end i to end j

$$X_i := \begin{bmatrix} x_{\left[(e_i)_2\right]} - x_{\left[(e_i)_1\right]} \\ y_{\left[(e_i)_2\right]} - y_{\left[(e_i)_1\right]} \end{bmatrix} \qquad X_3 = \begin{pmatrix} -6 \\ -3.464 \end{pmatrix} m \qquad L_i := \left| X_i \right|$$

Direction cosines

cosine sine

$$c_i := \frac{(X_i)_1}{L_i} \qquad s_i := \frac{(X_i)_2}{L_i}$$

$$T_i := \begin{pmatrix} c_i & s_i & 0 & 0 \\ 0 & 0 & c_i & s_i \end{pmatrix} \qquad T_1 = \begin{pmatrix} -0.707 & 0.707 & 0 & 0 \\ 0 & 0 & -0.707 & 0.707 \end{pmatrix}$$

Member stiffness matrix

Beam members

$$k_i := \frac{A_i \cdot E_i}{L_i} \cdot \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \qquad k_1 = \begin{pmatrix} 70710.68 & -70710.68 \\ -70710.68 & 70710.68 \end{pmatrix} \frac{kN}{m}$$

$$k_2 = \begin{pmatrix} 6.667 \times 10^4 & -6.667 \times 10^4 \\ -6.667 \times 10^4 & 6.667 \times 10^4 \end{pmatrix} \frac{kN}{m} \qquad k_3 = \begin{pmatrix} 4.33 \times 10^4 & -4.33 \times 10^4 \\ -4.33 \times 10^4 & 4.33 \times 10^4 \end{pmatrix} \frac{kN}{m}$$

Calculate element stiffness in global coordinates.

$$K_i := T_i^T \cdot k_i \cdot T_i \qquad K_1 = \begin{pmatrix} 35355.3 & -35355.3 & -35355.3 & 35355.3 \\ -35355.3 & 35355.3 & 35355.3 & -35355.3 \\ -35355.3 & 35355.3 & 35355.3 & -35355.3 \\ 35355.3 & -35355.3 & -35355.3 & 35355.3 \end{pmatrix} \frac{kN}{m}$$

$$K_2 = \begin{pmatrix} 66666.7 & 0 & -66666.7 & 0 \\ 0 & 0 & 0 & 0 \\ -66666.7 & 0 & 66666.7 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \frac{kN}{m} \qquad K_3 = \begin{pmatrix} 32476 & 18750 & -32476 & -18750 \\ 18750 & 10825.3 & -18750 & -10825.3 \\ -32476 & -18750 & 32476 & 18750 \\ -18750 & -10825.3 & 18750 & 10825.3 \end{pmatrix} \frac{kN}{m}$$

Zero out the global stiffness matrix.

$$S := \left| \begin{array}{l} \text{for } i \in 1..DOF \\ \quad \text{for } j \in 1..DOF \\ \qquad k_{init_{i,j}} \leftarrow 0 \\ k_{init} \end{array} \right.$$

$$S = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Assemble global structural stiffness matrix.

$$S := \begin{bmatrix} (K_1)_{1,1} + (K_2)_{1,1} + (K_3)_{1,1} & (K_1)_{1,2} + (K_2)_{1,2} + (K_3)_{1,2} & (K_1)_{1,3} & (K_1)_{1,4} & (K_2)_{1,3} & (K_2)_{1,4} & (K_3)_{1,3} & (K_3)_{1,4} \\ (K_1)_{2,1} + (K_2)_{2,1} + (K_3)_{2,1} & (K_1)_{2,2} + (K_2)_{2,2} + (K_3)_{2,2} & (K_1)_{2,3} & (K_1)_{2,4} & (K_2)_{2,3} & (K_2)_{2,4} & (K_3)_{2,3} & (K_3)_{2,4} \\ (K_1)_{3,1} & (K_1)_{3,2} & (K_1)_{3,3} & (K_1)_{3,4} & 0 & 0 & 0 & 0 \\ (K_1)_{4,1} & (K_1)_{4,2} & (K_1)_{4,3} & (K_1)_{4,4} & 0 & 0 & 0 & 0 \\ (K_2)_{3,1} & (K_2)_{3,2} & 0 & 0 & (K_2)_{3,3} & (K_2)_{3,4} & 0 & 0 \\ (K_2)_{4,1} & (K_2)_{4,2} & 0 & 0 & (K_2)_{4,3} & (K_2)_{4,4} & 0 & 0 \\ (K_3)_{3,1} & (K_3)_{3,2} & 0 & 0 & 0 & 0 & (K_3)_{3,3} & (K_3)_{3,4} \\ (K_3)_{4,1} & (K_3)_{4,2} & 0 & 0 & 0 & 0 & (K_3)_{4,3} & (K_3)_{4,4} \end{bmatrix}$$

Add known nodal forces

$$P = \begin{pmatrix} 50 \\ -100 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{pmatrix} \cdot kN$$

Therefore

$$P_1 := 50 \cdot kN$$

$$P_2 := -100 \cdot kN$$

Assemble non-nodal force matrices.

Members 1 and 2 have no non-nodal forces; member 3 has a temperature load.
In local coordinates:

$$Q_{F_1} := E_1 \cdot \alpha_1 \cdot A_1 \cdot \Delta T \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad Q_{F_1} = \begin{pmatrix} 324 \\ -324 \end{pmatrix} kN$$

$$Q_{F_2} := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot kN \qquad Q_{F_3} := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot kN$$

Transform non-nodal forces to global coordinates

$$F_{F_i} := {T_i}^T \cdot Q_{F_i}$$

$$F_{F_1} = \begin{pmatrix} -229.103 \\ 229.103 \\ 229.103 \\ -229.103 \end{pmatrix} kN \quad F_{F_2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} kN \quad F_{F_3} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} kN$$

Assemble non-nodal force matrix

$$P_F := \begin{bmatrix} (F_{F_1})_1 + (F_{F_2})_1 + (F_{F_3})_1 \\ (F_{F_1})_2 + (F_{F_2})_2 + (F_{F_3})_2 \\ (F_{F_1})_3 \\ (F_{F_1})_4 \\ (F_{F_2})_3 \\ (F_{F_2})_4 \\ (F_{F_3})_3 \\ (F_{F_3})_4 \end{bmatrix} \qquad P_F = \begin{pmatrix} -229.103 \\ 229.103 \\ 229.103 \\ -229.103 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} kN$$

Displacement degrees of freedom 3-8 are supported.

Extract the reduced stiffness matrix.

$$S_{red} := \text{submatrix}(S, 1, 2, 1, 2) \qquad S_{red} = \begin{pmatrix} 1.3 \times 10^5 & -16605.3 \\ -16605.3 & 46180.7 \end{pmatrix} \frac{kN}{m}$$

Extract the reduced nodal force matrix

$$P_{red} := \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} \qquad P_{red} = \begin{pmatrix} 50 \\ -100 \end{pmatrix} kN$$

Extract the reduced non-nodal force matrix.

$$P_{Fred} := \begin{pmatrix} P_{F_1} \\ P_{F_2} \end{pmatrix} \qquad P_{Fred} = \begin{pmatrix} -229.103 \\ 229.103 \end{pmatrix} kN$$

Calculate displacements

$$d_{red} := S_{red}^{-1} \cdot \left(P_{red} - P_{Fred}\right) \qquad d_{red} = \begin{pmatrix} 1.251 \\ -6.677 \end{pmatrix} mm$$

Reassemble structural displacement vector.

$$d := \begin{pmatrix} d_{red_1} \\ d_{red_2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Calculate the support reactions

$$P := S \cdot d + P_F \qquad P = \begin{pmatrix} 50 \\ -100 \\ -51.176 \\ 51.176 \\ -83.389 \\ 0 \\ 84.565 \\ 48.824 \end{pmatrix} kN$$

Member displacements

For element 1

$$v_1 := \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix} \qquad v_1 = \begin{pmatrix} 1.251 \\ -6.677 \\ 0 \\ 0 \end{pmatrix} mm$$

Element 2

$$v_2 := \begin{pmatrix} d_1 \\ d_2 \\ d_5 \\ d_6 \end{pmatrix} \qquad v_3 := \begin{pmatrix} d_1 \\ d_2 \\ d_7 \\ d_8 \end{pmatrix}$$

## Member forces in local coordinates

$$Q_i := k_i \cdot T_i \cdot v_i + Q_{F_i}$$

$$Q_1 = \begin{pmatrix} -72.374 \\ 72.374 \end{pmatrix} kN \qquad Q_2 = \begin{pmatrix} -83.389 \\ 83.389 \end{pmatrix} kN \qquad Q_3 = \begin{pmatrix} 97.647 \\ -97.647 \end{pmatrix} kN$$