

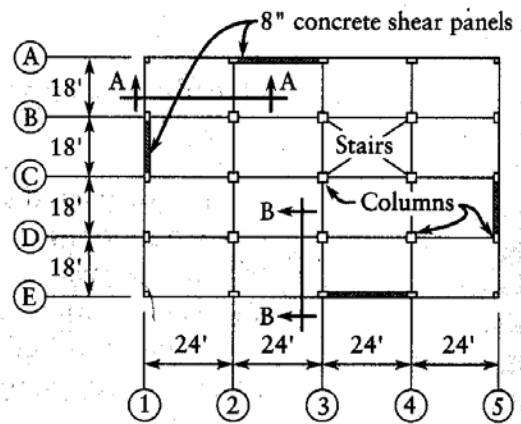
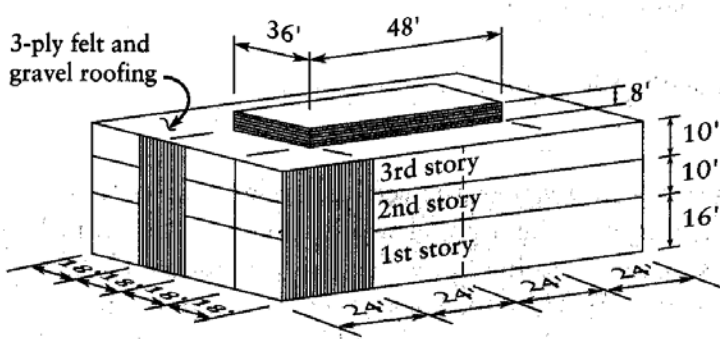
EXAM #1 - CE 342 - Fall 2000

Please use additional paper when necessary. Do not try to cram everything on the pages provided and do not write on the back of the pages.

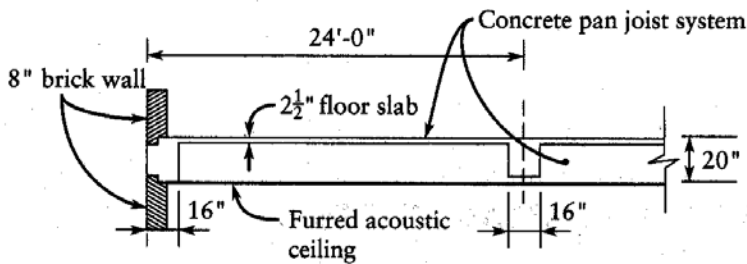
Problem #1. (25 points)

The office building shown below is being design to be built in Sandpoint, Idaho. The flat roof snow load calculated for this location is 63 psf. There are no adjacent structures and parapet walls are not high enough to cause drifting; however the penthouse can cause drifts on the third story roof. The drift load in the E-W direction is 34 psf for windward drifts and 96 psf for leeward drifts. The horizontal width of the windward drift is 5.0 ft; the horizontal width of the leeward drift is 14.2 ft. The drift load in the N-S direction is 20 psf for windward drifts and 84 psf for leeward drifts. The horizontal width of the windward drift is 4.3 ft; the horizontal width of the leeward drift is 12.4 ft.

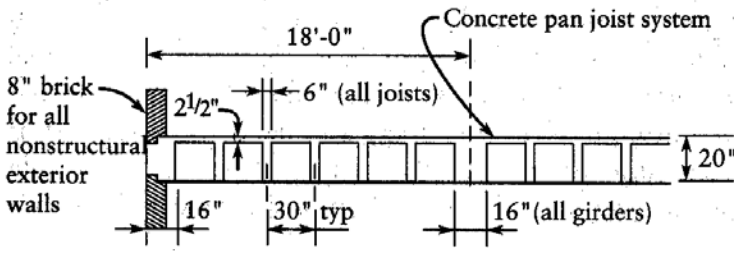
Determine the snow loads to be considered in the design of the roof joists in the penthouse and between grid lines A and B and grid lines 2 and 4 for the third-story roof. Ignore ponding and rain-on-snow surcharges. Assume the concrete framing is cast-in-place.



TYPICAL PLAN



SECTION A-A



SECTION B-B

Dead loads at each floor (and roof)

Dead load of roof-mounted equipment
43,200 lb

Dead load of roof screen wall
20,200 lb

Dead load of stairs and landings
50 lb/ft² (avg)

Allow 5 psf for dead load of interior partitions

Allow 3 psf for electrical dead load

Allow 4 psf for HVAC dead load

General notes

Occupancy: general office building

All columns are 16" square

Ground floor diaphragm is slab on grade

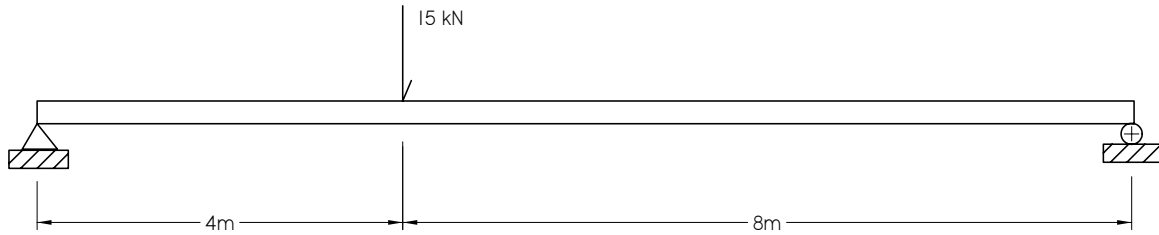
Interior footings are spread footings

Perimeter footings are grade beams

Roof live load is half the floor load

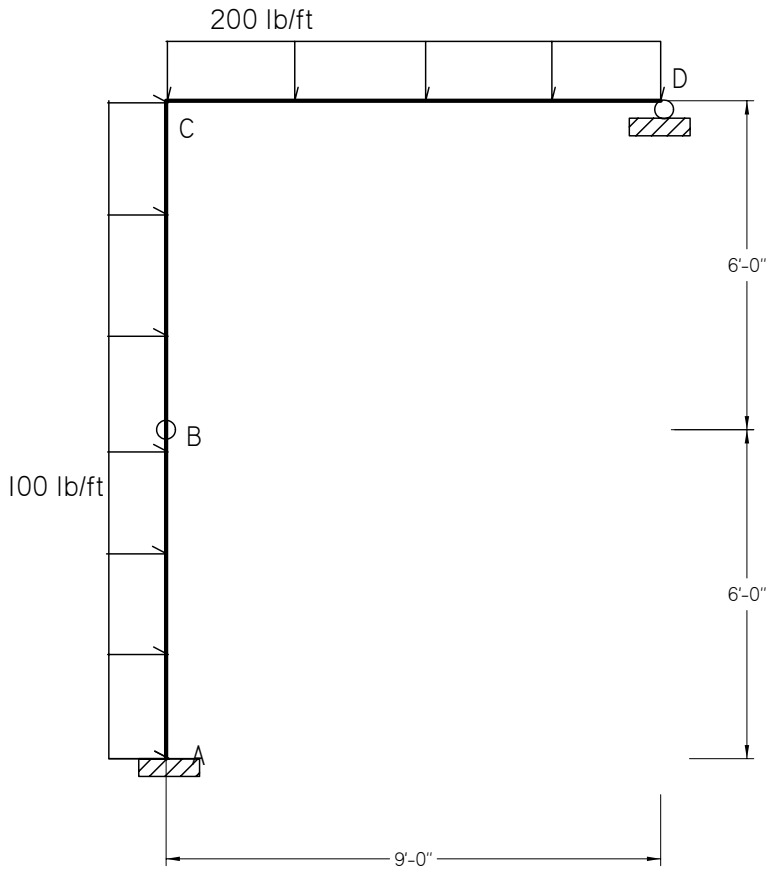
Problem #3. (25 points)

Find the maximum displacement in the beam shown below. Use the integration method. Assume $E = 200 \text{ GPa}$ and $I = 1000 \text{ mm}^4$.



Problem #4. (25 points)

Draw the shear, moment and axial force diagrams for the frame shown below.



Snow load calculations using IBC 2000
RJN 9/13/02

Exam problem Fall 2000

$$\text{psf} := \frac{\text{lbf}}{\text{ft}^2}$$

$$\text{kip} := 1000 \cdot \text{lbf}$$

Determine ground snow load

From snow load map

$$\text{ngsl} := 0.05 \cdot \frac{\text{lbf}}{\text{ft}^3}$$

from Topozone.com

$$\text{elev} := 2000 \cdot \text{ft}$$

$$p_g := \text{ngsl} \cdot \text{elev} \quad p_g = 100 \text{ psf}$$

Importance factor (Table 1604.5)

$$I_s := 1.0$$

Exposure Factor (Table 1608.3.1)

Based on the site description, use Exposure C, and assume the roof is fully exposed.

$$C_e := 0.9$$

Thermal factor (Table 1608.3.2)

Since roof is flat and there is no attic space, assume the roof is unventilated. Office space can be assumed to be heated. The R-value does not exceed 25.

$$C_t := 1.0$$

Flat roof snow load

$$p_f := 0.7 \cdot C_e \cdot C_t \cdot I_s \cdot p_g \quad p_f = 63 \text{ psf}$$

Rain-on-snow surcharge is ignored

Base snow load

Joists

Tributary width

$$s_j := 30 \cdot \text{in}$$

$$w_j := s_j \cdot p_f \quad w_j = 157.5 \frac{\text{lbf}}{\text{ft}}$$

Interior Beams - grid lines B, C & D, except at penthouse.

Tributary width

$$s_b := 33 \cdot \text{in}$$

$$w_b := s_b \cdot p_f \quad w_b = 173.25 \frac{\text{lbf}}{\text{ft}}$$

Exterior Beams - grid lines A & E, and B & D at penthouse.

$$s_{be} := \frac{s_b}{2}$$

The tributary width for the beams was not shown in the problem statement as assigned. It was corrected to make the dimensions add up, but it will not be considered in your homework score.

$$w_{be} := s_{be} \cdot p_f \quad w_{be} = 86.625 \frac{\text{lbf}}{\text{ft}}$$

Interior Girders, penthouse grid line 3 & roof level grid lines 2-4

$$s_g := 24 \cdot \text{ft}$$

$$w_g := s_g \cdot p_f \quad w_g = 1512 \frac{\text{lbf}}{\text{ft}}$$

Exterior Girders, penthouse grid lines 2 & 4, roof level grid lines 1 & 5

$$s_{ge} := 12 \cdot \text{ft}$$

$$w_{ge} := s_{ge} \cdot p_f \quad w_{ge} = 756 \frac{\text{lbf}}{\text{ft}}$$

Drift load

Drift load caused by penthouse on lower roof

Calculate density of drift. Ignore rain-on-snow while calculating drift loads.

$$\gamma := \frac{0.13}{\text{ft}} \cdot p_g + 14 \cdot \frac{\text{lbf}}{\text{ft}^3} \quad \gamma = 27 \frac{\text{lbf}}{\text{ft}^3}$$

If γ exceeds 30 psf, use 30 psf

$$\gamma := \text{if} \left(\gamma > 30 \cdot \frac{\text{lbf}}{\text{ft}^3}, 30 \cdot \frac{\text{lbf}}{\text{ft}^3}, \gamma \right) \quad \gamma = 27 \frac{\text{lbf}}{\text{ft}^3}$$

Calculate depth of base snow

$$h_b := \frac{p_f}{\gamma} \quad h_b = 2.333 \text{ ft}$$

Input height of obstruction (in this case, the penthouse wall)

$$h := 8 \cdot \text{ft}$$

Calculate height of cornice

$$h_c := h - h_b \quad h_c = 5.667 \text{ ft}$$

If base snow "buries" the cornice, ignore drift loads.

$$\frac{h_c}{h_b} = 2.429 \quad \text{If } h_c/h_b < 0.2, \text{ ignore drift loads}$$

Wind in long direction of building (East-West)

Calculate height of drifted snow.

Leeward direction

$$l_{u,lee} := 48 \cdot \text{ft}$$

$$h_{d,lee} := \left[\left[\frac{0.43}{3 \sqrt{\text{ft}}} \cdot \sqrt[3]{l_{u,lee}} \cdot \left(\frac{1}{4 \sqrt{\text{psf}}} \cdot \sqrt[4]{p_g + 10 \cdot \text{psf}} \right) \right] \right] \cdot \text{ft} - 1.5 \cdot \text{ft}$$

$$h_{d,lee} = 3.561 \text{ ft}$$

$$h_{d,EW} := h_{d,lee}$$

Windward direction

$$l_{u,w} := 24 \cdot \text{ft} \quad \text{but } l_{u,w} \text{ must be } \geq 25 \text{ ft.} \quad l_{u,w} := \text{if}(l_{u,w} < 25 \cdot \text{ft}, 25 \cdot \text{ft}, l_{u,w}) \quad l_{u,w} = 25 \text{ ft}$$

$$h_{d,w} := \frac{3}{4} \cdot \left[\frac{0.43}{\sqrt[3]{\text{ft}}} \cdot \sqrt[3]{l_{u,w}} \cdot \left(\frac{1}{\sqrt[4]{\text{psf}}} \cdot \sqrt[4]{p_g + 10 \cdot \text{psf}} \right) \right] \cdot \text{ft} - 1.5 \cdot \text{ft} \quad \text{The 2003 IBC increased this to } 3/4\text{'s of } h_d \text{ rather than } 1/2.$$

$$h_{d,w} = 1.929 \text{ ft}$$

$$h_{wd,EW} := h_{d,w}$$

Since wind can come from either direction, the windward and leeward directions can be reversed. This drift load can be applied to either the East or West wall.

Wind in short direction of building (North-South)

Leeward direction

$$l_{u,lee} := 36 \cdot \text{ft} \quad \text{but } l_{u,lee} \text{ must be } \geq 25 \text{ ft.} \quad l_{u,lee} := \text{if}(l_{u,lee} < 25 \cdot \text{ft}, 25 \cdot \text{ft}, l_{u,lee}) \quad l_{u,lee} = 36 \text{ ft}$$

$$h_{d,lee} := \left[\frac{0.43}{\sqrt[3]{\text{ft}}} \cdot \sqrt[3]{l_{u,lee}} \cdot \left(\frac{1}{\sqrt[4]{\text{psf}}} \cdot \sqrt[4]{p_g + 10 \cdot \text{psf}} \right) \right] \cdot \text{ft} - 1.5 \cdot \text{ft}$$

$$h_{d,lee} = 3.098 \text{ ft}$$

$$h_{d,NS} := h_{d,lee}$$

Windward direction

$$l_{u,w} := 18 \cdot \text{ft} \quad \text{but } l_u \text{ must be } \geq 25 \text{ ft.} \quad l_{u,w} := \text{if}(l_{u,w} < 25 \cdot \text{ft}, 25 \cdot \text{ft}, l_{u,w}) \quad l_{u,w} = 25 \text{ ft}$$

$$h_{d,w} := \frac{3}{4} \cdot \left[\frac{0.43}{\sqrt[3]{\text{ft}}} \cdot \sqrt[3]{l_{u,w}} \cdot \left(\frac{1}{\sqrt[4]{\text{psf}}} \cdot \sqrt[4]{p_g + 10 \cdot \text{psf}} \right) \right] \cdot \text{ft} - 1.5 \cdot \text{ft} \quad \text{The 2003 IBC increased this to } 3/4\text{'s of } h_d \text{ rather than } 1/2.$$

$$h_{d,w} = 1.929 \text{ ft}$$

$$h_{wd,NS} := h_{d,w}$$

Since wind can come from either direction, the windward and leeward directions can be reversed. This drift load can be applied to either wall.

Calculate drift load surcharge

Leeward direction

Long direction of building

$$p_{d,EW} := h_{d,EW} \cdot \gamma$$

$$p_{d,EW} = 96.145 \text{ psf}$$

Short direction of building

$$p_{d,NS} := h_{d,NS} \cdot \gamma$$

$$p_{d,NS} = 83.65 \text{ psf}$$

But drift load is ignored if base snow "buries" the cornice

$$p_{d,EW} := \text{if} \left(\frac{h_c}{h_b} < 0.2, 0 \cdot \text{psf}, p_{d,EW} \right)$$

$$p_{d,EW} = 96.145 \text{ psf}$$

$$p_{d,NS} := \text{if} \left(\frac{h_c}{h_b} < 0.2, 0 \cdot \text{psf}, p_{d,NS} \right)$$

$$p_{d,NS} = 83.65 \text{ psf}$$

Windward direction

Long direction of building

$$p_{wd,EW} := h_{wd,EW} \cdot \gamma$$

$$p_{wd,EW} = 52.081 \text{ psf}$$

Short direction of building

$$p_{wd,NS} := h_{wd,NS} \cdot \gamma$$

$$p_{wd,NS} = 52.081 \text{ psf}$$

But drift load is ignored if base snow "buries" the cornice

$$p_{wd,EW} := \text{if} \left(\frac{h_c}{h_b} < 0.2, 0 \cdot \text{psf}, p_{wd,EW} \right)$$

$$p_{wd,EW} = 52.081 \text{ psf}$$

$$p_{wd,NS} := \text{if} \left(\frac{h_c}{h_b} < 0.2, 0 \cdot \text{psf}, p_{wd,NS} \right)$$

$$p_{wd,NS} = 52.081 \text{ psf}$$

Calculate width of leeward drifts

Long direction of building

If h_d doesn't bury cornice

$$w_{1,EW} := 4 \cdot h_{d,EW}$$

$$w_{1,EW} = 14.244 \text{ ft}$$

Short direction of building

$$w_{1,NS} := 4 \cdot h_{d,NS}$$

$$w_{1,NS} = 12.393 \text{ ft}$$

If h_d buries cornice

$$w_{2,EW} := 4 \cdot \frac{h_{d,EW}^2}{h_c}$$

$$w_{2,EW} = 8.951 \text{ ft}$$

$$w_{2,NS} := 4 \cdot \frac{h_{d,NS}^2}{h_c}$$

$$w_{2,NS} = 6.775 \text{ ft}$$

Determine which case applies

$$w_{EW} := \text{if} (h_{d,EW} > h_c, w_{2,EW}, w_{1,EW})$$

$$w_{EW} = 14.244 \text{ ft}$$

$$w_{NS} := \text{if} (h_{d,NS} > h_c, w_{2,NS}, w_{1,NS})$$

$$w_{NS} = 12.393 \text{ ft}$$

If width of drift exceeds the width of the lower roof, truncate the drift at the far edge. (In this case, it doesn't).

Calculate width of windward drifts

Long direction of building

If h_d doesn't bury cornice

$$w_{w1,EW} := 4 \cdot h_{wd,EW}$$

$$w_{w1,EW} = 7.716 \text{ ft}$$

Short direction of building

$$w_{w1,NS} := 4 \cdot h_{wd,NS}$$

$$w_{w1,NS} = 7.716 \text{ ft}$$

If h_d buries cornice

$$w_{w2,EW} := 4 \cdot \frac{h_{wd,EW}^2}{h_c}$$

$$w_{w2,EW} = 2.626 \text{ ft}$$

$$w_{w2,NS} := 4 \cdot \frac{h_{wd,NS}^2}{h_c}$$

$$w_{w2,NS} = 2.626 \text{ ft}$$

Determine which case applies

$$w_{wEW} := \text{if} (h_{wd,EW} > h_c, w_{w2,EW}, w_{w1,EW})$$

$$w_{wEW} = 7.716 \text{ ft}$$

$$w_{wNS} := \text{if} (h_{wd,NS} > h_c, w_{w2,NS}, w_{w1,NS})$$

$$w_{wNS} = 7.716 \text{ ft}$$

If width of drift exceeds the width of the lower roof, truncate the drift at the far edge. (In this case, it doesn't).

Design loads for drift case

Joists spanning between grid lines 2, 3 & 4

Maximum drift snow load on joists nearest grid lines B & D (slightly conservative)

$$w_{jd} := s_j \cdot P_{d,NS} \quad w_{jd} = 209.126 \frac{\text{lbf}}{\text{ft}}$$

Total snow load

$$w_{jt} := w_{jd} + w_j \quad w_{jt} = 366.626 \frac{\text{lbf}}{\text{ft}}$$

Apply the new numbers to this sketch

Beams on grid lines B & D similarly carry the maximum drift load on half their tributary widths between grids lines 2-4.

$$w_{bd} := \frac{s_b}{2} \cdot P_{d,NS} \quad w_{bd} = 115.019 \frac{\text{lbf}}{\text{ft}}$$

Total snow load

$$w_{bt} := w_{bd} + w_b \quad w_{bt} = 201.644 \frac{\text{lbf}}{\text{ft}}$$

Joists spanning between grid lines 1 & 2 and between grid lines B & D have drift loads from the E-W drifts.

The maximum drift snow load is

$$w_{jd,EW} := s_j \cdot P_{d,EW} \quad w_{jd,EW} = 240.363 \frac{\text{lbf}}{\text{ft}}$$

Total snow load

$$w_{jt} := w_{jd,EW} + w_j \quad w_{jt} = 397.863 \frac{\text{lbf}}{\text{ft}}$$

Tapering down to the base snow load of $w_j = 157.5 \frac{\text{lbf}}{\text{ft}}$ at a distance $w_{EW} = 14.244 \text{ ft}$
On the windward side

$$w_{jwd,EW} := s_j \cdot P_{wd,EW} \quad w_{jwd,EW} = 130.202 \frac{\text{lbf}}{\text{ft}}$$

Total snow load

$$w_{jwt} := w_{jwd,EW} + w_j \quad w_{jwt} = 287.702 \frac{\text{lbf}}{\text{ft}}$$

Tapering down to the base snow load of $w_j = 157.5 \frac{\text{lbf}}{\text{ft}}$ at a distance $w_{wEW} = 7.716 \text{ ft}$

The beam spanning between grid lines 1 & 2 along grid line C has a similar drift load with

Maximum drift load

$$w_{bd,EW} := s_b \cdot P_{d,EW} \quad w_{bd,EW} = 264.399 \frac{\text{lbf}}{\text{ft}}$$

Total snow load

$$w_{bt} := w_{bd,EW} + w_b \quad w_{bt} = 437.649 \frac{\text{lbf}}{\text{ft}}$$

Tapering down to the base snow load of $w_b = 173.25 \frac{\text{lbf}}{\text{ft}}$ at a distance $w_{EW} = 14.244 \text{ ft}$

On the windward side

$$w_{\text{bwd.EW}} := S_b \cdot P_{\text{wd.EW}} \quad w_{\text{bwd.EW}} = 143.222 \frac{\text{lbf}}{\text{ft}}$$

Total snow load

$$w_{\text{bwt}} := w_{\text{jwd.EW}} + w_b \quad w_{\text{bwt}} = 303.452 \frac{\text{lbf}}{\text{ft}}$$

Tapering down to the base snow load of $w_j = 157.5 \frac{\text{lbf}}{\text{ft}}$ at a distance $w_{wEW} = 7.716 \text{ ft}$

Since the wind can reverse direction, the joist and beam snow loads should be "mirrored."

Drift load for girders on grid line 3 between

The maximum drift snow load is

$$w_{\text{gd.NS}} := S_g \cdot P_{\text{d.NS}} \quad w_{\text{gd.NS}} = 2007.6 \frac{\text{lbf}}{\text{ft}}$$

Total snow load

$$w_{\text{gt}} := w_{\text{gd.NS}} + w_g \quad w_{\text{gt}} = 3519.6 \frac{\text{lbf}}{\text{ft}}$$

Tapering down to the base snow load of $w_g = 1512 \frac{\text{lbf}}{\text{ft}}$ at a distance $w_{NS} = 12.393 \text{ ft}$

On the windward side

$$w_{\text{gwd.NS}} := S_g \cdot P_{\text{wd.NS}} \quad w_{\text{gwd.NS}} = 1.25 \times 10^3 \frac{\text{lbf}}{\text{ft}}$$

Total snow load

$$w_{\text{gwt}} := w_{\text{gwd.NS}} + w_g \quad w_{\text{gwt}} = 2761.9 \frac{\text{lbf}}{\text{ft}}$$

Tapering down to the base snow load of $w_g = 1512 \frac{\text{lbf}}{\text{ft}}$ at a distance $w_{wNS} = 7.716 \text{ ft}$

Girders on grid lines 2 & 4 between grid lines A & B and D & E only have a drift load on half their tributary width since there is no drifting between grid lines 1 & 2 and 4 & 5.

The maximum drift snow load is

$$w_{\text{gd.NS}} := \frac{S_g}{2} \cdot P_{\text{d.EW}} \quad w_{\text{gd.NS}} = 1.154 \times 10^3 \frac{\text{lbf}}{\text{ft}}$$

Total snow load

$$w_{\text{gt}} := w_{\text{gd.NS}} + w_g \quad w_{\text{gt}} = 2665.7 \frac{\text{lbf}}{\text{ft}}$$

Tapering down to the base snow load of $w_g = 1512 \frac{\text{lbf}}{\text{ft}}$ at a distance $w_{NS} = 12.393 \text{ ft}$

On the windward side

$$w_{\text{gwd.NS}} := \frac{S_g}{2} \cdot P_{\text{wd.NS}} \quad w_{\text{gwd.NS}} = 624.971 \frac{\text{lbf}}{\text{ft}}$$

Total snow load

$$w_{\text{gwt}} := w_{\text{gwd.NS}} + w_g \quad w_{\text{gwt}} = 2137 \frac{\text{lbf}}{\text{ft}}$$

Tapering down to the base snow load of $w_g = 1512 \frac{\text{lbf}}{\text{ft}}$ at a distance $w_{wNS} = 7.716 \text{ ft}$

Girders on grid lines 2 & 4 also have drift loads from the joists spanning between grid lines 1 & 2 and 4 & 5.

As with the tributary area calculations, assume the joists are simply supported. From the drift load calculations for joists spanning from grid line 1 to 2 we have

Summing moments about grid line 1

$$R_2 \cdot 24 \cdot \text{ft} - w_j \cdot 24 \cdot \text{ft} \cdot \frac{24}{2} \cdot \text{ft} - \frac{1}{2} \cdot w_{EW} \cdot w_{jd.EW} \cdot \left(24 \cdot \text{ft} - \frac{1}{3} \cdot w_{EW} \right) = 0$$

Solving for R_2

$$R_2 := \frac{1}{144} \cdot \frac{1728 \cdot w_j \cdot \text{ft}^2 + 72 \cdot w_{EW} \cdot w_{jd.EW} \cdot \text{ft} - w_{EW}^2 \cdot w_{jd.EW}}{\text{ft}} \quad R_2 = 3263.2 \text{ lbf}$$

Distributing this over the joist's tributary width

$$w_{gdBD} := \frac{R_2}{s_j} \quad w_{gdBD} = 1.305 \times 10^3 \frac{\text{lbf}}{\text{ft}}$$

Total load

$$w_{gtBD} := w_{gdBD} + w_{ge} \quad w_{gtBD} = 2061.3 \frac{\text{lbf}}{\text{ft}}$$

Drift loads due to parapet walls

Recall that the height of the base snow is:

$$h_b = 2.333 \text{ ft}$$

Since the parapet wall is only two feet tall, it is "buried" by the base snow load, and no drifts are formed.

The load equation is:

$$EIy'''' = w(x) = -15\langle x - 4 \rangle^{-1}$$

Integrating to find the shear equation

$$EIy''' = S(x) = -15\langle x - 4 \rangle^0 + C_1$$

From statics, we know that the reaction at the left-hand end is 10 kN upwards. Using this as a boundary condition

$$S(0) = 10 = -15 \cdot 0 + C_1$$
$$C_1 = 10$$

Integrating to find the moment equation

$$EIy'' = M(x) = -15\langle x - 4 \rangle^1 + 10x + C_2$$

From statics, we know that the moment at the left-hand end is zero. Using this as a boundary condition

$$M(0) = 0 = -15 \cdot 0 + 10 \cdot 0 - C_2$$
$$C_2 = 0$$

Integrating to find the slope equation

$$EIy' = q(x) = -\frac{15}{2}\langle x - 4 \rangle^2 + \frac{10}{2}x^2 + C_3$$

Integrating to find the displacement equation

$$EIy = -\frac{5}{2}\langle x - 4 \rangle^3 + \frac{5}{3}x^3 + C_3x + C_4$$

Displacement boundary conditions are $y(0) = y(12) = 0$

$$EIy(0) = 0 = -\frac{5}{2} \cdot 0 + \frac{5}{3} \cdot 0 + C_3 \cdot 0 + C_4$$
$$C_4 = 0$$

and

$$EIy(12) = 0 = -\frac{5}{2}(8)^3 + \frac{5}{3}(12)^3 + C_3 \cdot 12$$

$$C_3 = -133.33$$

The slope and displacement equations are thus:

$$q(x) = \frac{1}{EI} \left[-\frac{15}{2} \langle x-4 \rangle^2 + 5x^2 - 133.33 \right]$$

$$y(x) = \frac{1}{EI} \left[-\frac{5}{2} \langle x-4 \rangle^3 + \frac{5}{3} x^3 - 133.33x \right]$$

By inspection, we can see that the maximum displacement occurs to the right of the point load. Therefore we will assume that $x > 4$ m, and solve for the location that provides a zero slope (which is where the maximum displacement occurs) and then calculate the displacement at this location. Since we have assumed that $x > 4$ m, we can treat the discontinuity function as a “standard” polynomial. Solving for the point of zero slope

$$0 = \frac{1}{EI} \left[-\frac{15}{2} (x-4)^2 + 5x^2 - 133.33 \right]$$

$$0 = -7.5x^2 + 60x - 120 + 5x^2 - 133.33$$

$$0 = -2.5x^2 + 60x - 253.33$$

$$x^2 - 24x + 101.33 = 0$$

The roots to this equation are $x = 5.468$ m and $x = 18.53$ m. The second root is not on the beam, so we select the first root. The displacement at this location is

$$y(5.468) = \frac{1}{EI} \left[-\frac{5}{2} (5.468-4)^3 - \frac{5}{3} (5.468)^3 - 133.33(5.468) \right]$$

$$y_{\max} = \frac{464.5}{EI}$$

