Engineering Economics

CE 215

Time Value of Money

- Time affects the cost of money.
  - A dollar now is worth more than a dollar a year from now because it can earn interest during the year.
- Interest represents two things:
  - It is the "rent" someone pays for using our money for a period of time.
  - The amount of rent they pay should reflect the risk we take in making the loan.
    - Riskier loans require more interest to make them attractive.

History of Interest

- The concept of interest goes back to earliest recorded history.
  - Babylon 2000 B.C. – money paid for use of grain that was borrowed.
  - Typical rates were 6 to 25% per annum.
- Usury is prohibited in the Law of Moses, and in Islamic cultures.
- In the middle ages, interest on loans was prohibited based on these restrictions.

History of Interest (cont.)

- In 1536, John Calvin adopted a theory of what constituted usury that allowed interest.
- Islamic conventions developed to allow those with money to buy a stake in a business in return for an portion of the profit from that business.
- Interest and the cost of capital have become an essential part of doing business.

Simple vs. Compound Interest

- Simple interest charges are in direct proportion to the original amount of the loan.
- For a car loan of $6000, and a monthly interest rate of 1% (12% per year)
- If the car payments are $300/month, then
  - $60/month goes to pay interest, every month, and
  - $240/month goes towards the principle every month.
  - 25 payments for a total of payments of $7500, and
  - $1500 in interest payments.

Compound Interest

- If the same amount is loaned at the same monthly interest rate, but at compound interest, the interest payment is proportional to the balance at any point in time, arranged to be paid in 25 months.
- Interest payment is $60 for the first month, $57.88 the second … and $2.70 in the 25th month.
  - Total of payments is $6811 with
  - $811 in interest payments.

Compound Interest (cont.)

- Most major economic projects are financed with compound interest, and
- Our formulas will focus on compound interest.
  - They also assume that payments are made at the end of the time periods, similar to the car payment example just shown.

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Financing Alternatives

- The ability to account for the time value of money allows us to create a variety of financing schemes.
  - Constant payments on principle, plus interest accrued over time period
    - Results in decreasing payments since principle declines.
  - Interest-only payments each year, plus principle and interest at end of period.
  - Constant payments for each period, with interest amount declining over time.
  - Balloon payment at the end.

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Cash Flow Diagrams

- Cash flow diagrams illustrate the dollar transactions, including
  - The amount of each transaction,
  - When they take place,
  - What direction they travel.
- Cash flow diagrams are to accountants and financiers as free-body diagrams are to engineers.

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Equivalence

- The ability to vary interest payment schemes makes it very difficult to compare financing alternatives.
- The examples illustrated before are equivalent since they:
  - Have the same interest rate,
  - Are based on the same loan amount,
  - Involve the same time period.

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Bases of Comparison

- Economists and accountants have developed some standard definitions that can be used for comparisons between financing alternatives.
  - Present value – The present worth of a series of uniform payments.
  - Future value – The future worth of a series of uniform payments (i.e., the value at the end of the payments).
  - Sinking Fund – The annual/periodic payments that must be set aside to provide a specified future value.
  - Capital Recovery – The present worth of a series of annual/periodic payments.

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Interest Formulas

- Formulas can be derived to facilitate comparisons between the various forms of payments.
- These formulas assume:
  - Discrete interest payments – The payments occur at discrete time intervals, e.g., every year, or every month.
  - Interest payments are made at the end of each time interval.

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Interest Formulas (cont.)

- Formulas can be derived for
  - Continuous interest payments, or
  - Interest payments at the start of each time interval.
- However, these are less common and won't be considered here.

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Notation

- We will use the following notation:
  - i = effective interest rate per interest period,
  - N = number of compounding periods,
  - P = present sum of money,
  - F = future sum of money,
  - A = payments made at the end of each interest period.

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Present given Future

- How much should you invest now to have a future sum of money, given
  - i = 5% per annum
  - F = \$10,000
  - N = 15 years

$$P = F\frac{1}{(1+i)^N} = \frac{10{,}000}{(1.05)^{15}} = \$4810.17$$

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Mortgage Payments.

- Suppose you buy a house for \$120,000, with 10% down, 1 ½ points, 6% interest, with a 30-year loan. What will be your payments?
  - P = \$120,000 - 0.10(\$120,000) +0.015(\$108,000) = \$109,620
  - N = 30(12) = 360
  - i = 0.06/12 = 0.005

$$A = P\frac{i(1+i)^N}{(1+i)^N - 1} = \$109{,}620\frac{0.005(1.005)^{360}}{1.005^{360} - 1} = \$657.23$$

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Mortgage Payments (cont.)

- Suppose the interest rate on the previous loan was 10% per year. What are the monthly payments?
  - A = \$961.99
- Which explains why interest rates have been headline news for the past year or more.

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Finding Interest Rates

- Suppose you want to borrow \$120,000 less 10% down plus 1 ½ points, and feel that you can afford up to \$750 per month house payments. What is the maximum interest rate you can afford?

$$\$750 = \$109{,}620\frac{i(1+i)^{360}}{(1+i)^{360} - 1}$$

- This is a nonlinear function of i, which can be solved with Mathcad's root function, for example.
  - i = 7.3%

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Saving for Retirement

- Suppose you graduate at age 25 and plan to save $100/paycheck from 25 to 65, how much would you have accumulated at retirement, assuming
  - i = 10%/26 pay periods per year = 0.38%
  - N = 20 years(26 pay periods per year) = 1040
  - A = $100

$$F = A\frac{(1+i)^N - 1}{i} = \$100\frac{(1.00385)^{1040} - 1}{0.00385} = \$1{,}382{,}702$$

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Saving for Retirement (cont.)

- Suppose that sounds too hard to manage, what with student loans, etc. Instead, you plan to save $200/paycheck from age 45 to 65,
  - This results in the same total payment to the retirement plan.
- Assuming the same interest rate,

$$F = \$200\frac{(1.00385)^{520} - 1}{0.00385} = \$330{,}760$$

  - Which is less than *one-quarter* of the future value of the previous plan.

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Retirement Payout

- Suppose you plan to
  - Retire at age 65 with $1,000,000
  - Live from 65 to 85, on income from these assets,
  - Leave $200,000 for your estate.
- How much can be withdrawn each month?
  - Assuming an annual interest rate of 10%.
  - i = 10%/12 payments per year = 0.00833
  - N = 20 years(12 per year) = 240

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Retirement Payout (cont.)

- In order to compare $1,000,000 at retirement to $200,000 at the end of the term, calculate the present value (at retirement) of $200,000.

$$P = F(1+i)^{-N} = \$200{,}000(1.00833)^{-240} = \$27{,}292.30$$

- In other words if $27,292.30 of the amount at retirement is set aside for the 20 years of retirement, it will be worth $200,000 at the end of term.
  - The remainder $1,000,000 - 27,292.30 = $972,702.70 can be paid out as an annuity.

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Retirement Payout (concl.)

- The monthly payment from the retirement fund will be

$$A = P\frac{i(1+i)^N}{(1+i)^N - 1} = \$972{,}707.70\frac{0.00833(1.00833)^{240}}{(1.00833)^{240} - 1} = \$9386.84$$

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Rule of Seventy-two

- To compare investments, we often ask, “How long will it take for an investment to double in value.”
- The answer is often approximated by the “rule of 72.”
  - The time to double an investment is calculated by dividing 72 by the interest rate.
  - E.g., the time to double an investment at 12% interest is 72/12 = 6 years.

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Exact Version

- The exact answer can be calculated from

$$N = \frac{\log(2)}{\log(1+i)}$$

- For i = 12%,

$$N = \frac{\log(2)}{\log(1.12)} = 6.12 \text{ years}$$

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Summary

- The time-value of money is a vital component of business and personal finance.
- Most financing is arranged with compound interest, and compound interest is the basis of the calculations we will do.
- Cash-flow diagrams illustrate the cash that changes hands over the term of a transaction.

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Summary (cont.)

- A series of equations were provided to allow comparisons between financing arrangements.
- Most equations included a combination of interest rate, number of payments, present and future worth, and periodic payments.

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