



Independent Study IN IDAHO

Math 143

Pre-calculus Algebra and Analytic Geometry

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**The University of Idaho in statewide cooperation with
Boise State University — Idaho State University
Lewis-Clark State College**

Study Guide

Independent

Study IN IDAHO

PO Box 443225

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Self-paced study. Anytime. Anywhere!

Math 143

Pre-calculus Algebra and Analytic Geometry

University of Idaho

3 Semester-Hour Credits

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University of Idaho

RV: 5/06

2-Math 143

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Independent Study in Idaho

This course is offered by the University of Idaho.


Pre-calculus Algebra and Analytic Geometry

3 Semester-Hour Credits: UI

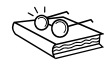
Welcome!

Whether you are a new or returning student, welcome to the Independent Study in Idaho (ISI) program. Before beginning this course, read the information provided below, including course description, prerequisites, required materials, course objectives, and information about lessons, exams, and grading.

Important!

As you read this section, you will see the following icon: 

Use this icon to direct yourself to the **Appendix** in the back of this study guide for essential registration information, Independent Study in Idaho policies and procedures, and forms you will need to successfully complete this course. You are responsible for understanding and following ISI policies and procedures.



Turn to the **Appendix** now. Familiarize yourself with the information in the *Registration* section, student responsibilities in *Academic Integrity*, and the necessary forms. If there is anything you do not understand, please contact the ISI office for clarification before starting your course.

Course Description

Algebraic, exponential, logarithmic functions; graphs of conics; zeros of polynomials; systems of equations, induction. UI Students: Carries no credit after Math 160 (Survey of Calculus) or Math 170 (Analytic Geometry and Calculus I); carries 2 credits after Math 137 (Algebra with Applications).

Prerequisites

1½ years high school algebra, 1 year high school plane geometry, and sufficient score on SAT, ACT, or COMPASS Math Test; or Math 108 (Intermediate Algebra) with grade of C or better. It is recommended that Math 143 be taken within two years of passing Math 108 or its equivalent.

Course Materials

Required Course Materials

- Stewart, James, Lothar Redlin, and Saleem Watson. *Precalculus: Mathematics for Calculus*. 5th ed. Belmont, CA: Thomson Brooks/Cole, 2006. ISBN: 0-534-49277-0.

Recommended Course Materials

- Stewart, James, Lothar Redlin, and Saleem Watson. *Student Solutions Manual for Stewart/Redlin/Watson's Precalculus: Mathematics for Calculus*. 5th ed. Belmont, CA: Thomson Brooks/Cole, 2006. ISBN: 0-534-49290-8.

Independent Study in Idaho course materials are available for purchase at the University of Idaho Bookstore. Visit the UI Bookstore's Web site, <http://www.uidahobookstore.com>, select *Textbook*, *Independent Study* for a list of course materials. You may order online, by telephone, (208) 885-7334, or by e-mail to uibooks@uidaho.edu.

Independent Study in Idaho courses are updated and revised periodically. Ordering course materials from the UI Bookstore at the time of registration allows you to purchase the correct edition(s) of textbooks, study guides, and supplemental materials. If purchasing textbooks from another source, refer to the ISBN(s) for the textbook(s) listed for this course to ensure that you obtain the correct edition(s). If you have questions regarding the course materials you have ordered and received, contact the UI Bookstore.

Course Introduction

The course has been divided into 16 lessons and four exams. Submit all lessons and exams for grading. Omit any chapters or sections not included in the assignments. You must wait for the graded assignments to be returned before taking the exams.

If you want more problems to solve to make sure you understand the material, there are several books available that have problems and solutions. *Schaum's Outline Series of College Algebra* is a good one and is available from the University of Idaho Bookstore.

Each of the lessons covered in this study guide is constructed with reading assignments in the text and practice problems. These alternate so that you read a selection and then work applicable practice problems. At the end of the lesson, there is a graded assignment section with a list of problems from the textbook, which you will submit for grading. These questions will cover a sample of the material. It is not possible to cover all of the material for each lesson in each graded assignment; therefore, the practice problems are very important. The grade on each assignment will be based on the work shown, as well as on the accuracy of the answer.

It is important that you understand the material presented before trying the graded assignments. Work through the examples in the reading assignments and the associated practice problems. The answers to all odd-numbered problems are located in the back of the textbook. If your answer differs from that in the back of the book, carefully check for errors and, if necessary, work the problem again. You will find that you usually need to write out more steps than the author does in the examples. The author sometimes skips steps in order to keep the number of pages to a minimum. Any section of the text not specifically mentioned in the lessons should be skipped (for example, Section 1.9 will be skipped, but Sections 1.8 and 1.10 will be covered).

There is a summary of important ideas, formulas, and rules for each lesson in the study guide. If you find a formula in the text that is not mentioned in the study guide, you do not need to memorize that formula for the exams.

Complete the graded assignments in **pencil only**. Copy the original question, show your work in a vertical format on the page, and circle the answer. Neatness and completeness are essential. You may use a nontext calculator on both assignments and exams, but very few problems will need them. Keep in mind that we are grading and testing your understanding of the material, not your ability to push buttons. Therefore, it is in your best interest not to rely on calculators while doing the assignments. Nevertheless, you *will* need a calculator for the sections on exponentials and logarithms.

Your instructor wishes you success in your study of pre-calculus and hopes that your experience with the course is both profitable and enjoyable.

Lessons

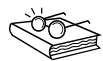
Overview

Each lesson includes the following components:

- Reading assignments
- Lesson objectives
- Practice problems
- Graded assignment

Study Hints

- Complete all assigned readings.
- Set a schedule allowing for completion of the course one month prior to your desired deadline.
(An *Assignment Submission Log* is provided for this purpose.)



See the **Appendix** at the back of this study guide for essential *ISI policies on submitting lessons to your instructor*. See the letters sent in your registration packet for *your instructor's requirements: how to format and submit lessons; number of lessons you may submit at one time, and lesson guidelines*.

Exams

Overview

- Submit all lessons preceding each exam prior to taking it. For each exam, your instructor strongly recommends that you wait until the lessons preceding the exam have been returned to you graded, with feedback, before taking the exam.
- For your instructor's exam guidelines, refer to the letters sent in your registration packet and the *Exam Information* sections in this study guide.

The timing of the exams follows:

- Exam 1 (Sections 1.5–2.2): follows Lesson 4.
- Exam 2 (Sections 2.3–3.2): follows Lesson 8.
- Exam 3 (Sections 3.3–4.4): follows Lesson 12.
- Final Exam (Sections 1.5–10.4): follows Lesson 16.

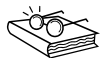
The first three exams will be 70 minutes long and will cover only the material studied since the previous exam. The exams will be made up of problems similar to the assigned problems (both practice problems and graded problems).

The final exam will be similar to the first three exams except that it will be two hours long and will cover the entire course. That is, it will cover all assigned sections, from 1.5 through 10.4.

See *Grading* for specific information on exams, points, and percentages.

Choosing a Proctor/Scheduling Exams

All exams require a proctor unless an exam is self-administered.



See the **Appendix** for guidelines on *how to choose a proctor and schedule exams*.

Grading

The course grade will be based upon the following considerations:

The grading for this course will be based on 1,300 points, broken down as follows:

Graded Assignments	300 points (20 points each, lowest assignment dropped)
Exams	600 points (200 points each)
Final Exam	400 points

A letter grade is assigned only at the end of the course, as follows:

A = 1170 – 1300	D = 780 – 909
B = 1040 – 1169	F = 0 – 779
C = 910 – 1039	

The final course grade is issued after **all** lessons and exams have been graded.



See the **Appendix** for information about *confidentiality of student grades, course completion and time considerations*, and *requesting a transcript*.

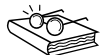
Contacting Your Instructor

You will receive *course and instructor contact information* in your registration packet.



See the **Appendix** for detailed information on *contacting your instructor*.

Disability Support Services



See the **Appendix** for information on *Disability Support Services (DSS)*.

Assignment Submission Log

Lesson	Section	Graded Assignment	Date Submitted
1	1.5	exercises 4, 14, 44, 50, 70, 80, 82, 90, 96, 100	
2	1.6 1.7	exercises 20, 40, 60, 70 exercises 16, 38, 58, 68, 74, 98	
3	1.8 1.10	exercises 8, 44, 60, 88, 92 exercises 20, 28, 34, 46, 72	
4	2.1 2.2	exercises 4, 16, 24, 36, 52 exercises 24, 34 (sketch by hand), 50, 60, 66	
It is time to make arrangements with your proctor to take Exam 1.			
5	2.3 2.4	exercises 2, 4, 16, 34 exercises 18, 32, 40, 64, 66, 68	
6	2.5 2.6	exercises 10, 28, 36, 40, 64 exercises 2, 10, 20, 24, 26	
7	2.7 2.8	exercises 4, 22, 26, 40, 46 exercises 8, 20, 28, 38, 54	
8	3.1 3.2	exercises 2, 10, 16, 26, 30 exercises 22, 36, 50, 56, 62	
It is time to make arrangements with your proctor to take Exam 2.			
9	3.3	exercises 12, 36, 40, 44, 50, 56, 58, 60, 66, 74	
10	3.4 3.5	exercises 10, 34, 54, 58, 66 exercises 8, 20, 40, 52, 60	
11	4.1 4.2	exercises 16, 30, 66, 78, 80 exercises 14, 32, 40, 54, 62	
12	4.3 4.4	exercises 6, 28, 42, 56 exercises 10, 22, 32, 42, 50, 68	
It is time to make arrangements with your proctor to take Exam 3.			

13	4.5	exercises 2, 6, 8, 10, 12, 14, 18, 20, 22, 24	
14	9.2 9.3	exercises 10, 18, 22, 26, 46, 48, 52, 56 exercises 16, 18	
15	10.1 10.2	exercises 12, 16, 32, 36, 40 exercises 14, 18, 24, 30, 38	
16	10.3 10.4	exercises 8, 12, 20, 28, 38 exercises 2, 8, 14, 16, 20	
It is time to make arrangements with your proctor to take the Final Exam.			

Lesson 1

Equations

Reading Assignments

Section 1.5

Lesson Objectives

Note: This is a good time to become familiar with the layout of the textbook. Each new section in the text has introductory material that the student should read very carefully. You will notice that the important words or topics will be in **boldface**. For example, on page 44 the words **equivalent equations** appear in bold type and are defined as two equations with exactly the same solution. You should be certain that you understand the meaning of all boldface words.

The items found in the blue boxes are another important feature of this textbook. Concepts found in the blue boxes are very important and should be written down by the student. For example, on page 49 you will find one of the most important formulas in all mathematics, the **Quadratic Formula**.

THE QUADRATIC FORMULA
<p>The roots of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are</p> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$

The examples are probably the most important aspect of each section. Each section contains several worked examples that are very similar to your homework problems. You should work through the examples very carefully and refer to them while doing your homework. Follow the directions in the lesson and then work odd-numbered practice problems related to the reading material. All odd-numbered problems have solutions in the back of the book.

Objectives for Section 1.5

- Know how to solve a linear equation $ax + b = 0$.

Example: $5x - 23 = 7x + 12$

$$\begin{aligned} -23 - 12 &= 7x - 5x \\ -35 &= 2x \\ \frac{-35}{2} &= x \end{aligned}$$

- Know that to solve an equation you can do the following:
 1. Add or subtract the same quantity from both sides.
 2. Multiply or divide both sides by the same nonzero quantity.

Note: The example $5x - 23 = 7x + 12$, given above, uses both 1 and 2.

3. Raise both sides of an equation to the same power. Examples are given on the next page.

- Know the definition of the **Quadratic Equation**.

$$ax^2 + bx + c = 0, \text{ where } a, b, \text{ and } c \text{ are real and } a \neq 0$$

- Know the **Zero-Product Property**.
- Be able to factor a quadratic or trinomial expression or equation. (See Section 1.3, pages 27–31, for a review of factoring.)

Examples: $x^2 - 3x - 28 = (x - 7)(x + 4)$

$$6x^2 + x - 12 = (2x + 3)(3x - 4)$$

- Be able to solve a simple quadratic equation by extracting square roots.
Note: You should use this method when $b = 0$.

Example: $2x^2 - 32 = 0$
 $2x^2 = 32$
 $x^2 = 16$
 $x = \pm 4$

- Be able to solve a quadratic equation by completing the square.

Example: $x^2 + 6x - 4 = 0$	
$x^2 + 6x = 4$	Add 4 to both sides.
$x^2 + 6x + 9 = 13$	Add $(\frac{6}{2})^2 = 9$ to both sides.
$(x + 3)^2 = 13$	Factor.
$x + 3 = \pm\sqrt{13}$	Take square roots of both sides.
$x = -3 \pm\sqrt{13}$	Subtract 3 from both sides.

- Know the Quadratic Formula and how to use it.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Be able to determine the types of solutions of a quadratic equation by checking the discriminant $b^2 - 4ac$.

1. If $b^2 - 4ac > 0$, then there are two distinct real solutions.

Example:
 $16x^2 + 8x - 3 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(16)(-3)}}{2(16)} = \frac{-8 \pm \sqrt{256}}{32} = \frac{-8 \pm 16}{32}$$

$$x = \frac{-8 + 16}{32} \text{ or } x = \frac{-8 - 16}{32}$$

$$x = \frac{1}{4} \text{ or } x = -\frac{3}{4}$$

2. If $b^2 - 4ac = 0$, then there is exactly one real solution.

Example:

$$4x^2 - 4x + 1 = 0$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(4)(1)}}{2(4)} = \frac{4 \pm \sqrt{16 - 16}}{8} = \frac{4 \pm \sqrt{0}}{8}$$

$$x = \frac{4}{8} = \frac{1}{2}$$

3. If $b^2 - 4ac < 0$, then there are no real solutions.

Example:

$$3x^2 + 6x + 5 = 0$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(5)}}{2(3)} = \frac{-6 \pm \sqrt{-24}}{6} = \frac{-6 \pm i\sqrt{24}}{6} = \frac{-6 \pm 2i\sqrt{6}}{6} = \frac{-3 \pm i\sqrt{6}}{3}$$

Here the solutions are complex numbers. Those will be covered in Lesson 10, Section 3.4.

- Be able to solve certain types of nonlinear or nonquadratic equations:

Example: $x^6 - 64 = 0$

Let $w = x^3$

$$w^2 = (x^3)^2 = x^6$$

$$w^2 - 64 = 0$$

$$(w - 8)(w + 8) = 0$$

$$w = 8 \text{ or } w = -8$$

$$x^3 = 8 \text{ or } x^3 = -8$$

$$\sqrt[3]{x^3} = \sqrt[3]{8} \text{ or } \sqrt[3]{x^3} = \sqrt[3]{-8}$$

$$x = 2 \text{ or } x = -2$$

Try substitution.

Factor by difference of squares. (See Section 1.3.)

Substitute back.

Take cube roots of both sides.

Example: $9x^{2/3} + 24x^{1/3} + 16 = 0$

$$9(x^{1/3})^2 + 24(x^{1/3}) + 16 = 0$$

$$9w^2 + 24w + 16 = 0$$

$$(3w + 4)(3w + 4) = 0$$

Let $w = x^{1/3}$.

Substitute.

Factor.

$$w = -\frac{4}{3}$$

$$x^{1/3} = -\frac{4}{3}$$

Substitute back.

$$\left(x^{1/3}\right)^3 = \left(-\frac{4}{3}\right)^3$$

Cube both sides.

$$x = -\frac{64}{27}$$

Example: $x^3 + 2x^2 - 3x - 6 = 0$

$$(x^3 + 2x^2) + (-3x - 6) = 0$$

Group.

$$x^2(x + 2) - 3(x + 2) = 0$$

$$(x^2 - 3)(x + 2) = 0$$

Factor by grouping.

$$(x + \sqrt{3})(x - \sqrt{3})(x + 2) = 0$$

Use the difference of squares with $x^2 - (\sqrt{3})^2$.

$$x = -\sqrt{3}, x = \sqrt{3}, x = -2$$

- For equations involving radicals or fractional powers, you may need to raise both sides to the same power. Always isolate the radical first.

Example: $\sqrt{5-x} - 3 = 0$

$$\sqrt{5-x} = 3$$

Isolate the radical.

$$(\sqrt{5-x})^2 = 3^2$$

Square both sides.

$$5-x = 9$$

Solve for x .

$$x = -4$$

Example: $\sqrt{x} - \sqrt{x-5} = 1$

There are two radicals that must be isolated in steps.

$$\sqrt{x} = \sqrt{x-5} + 1$$

$$(\sqrt{x})^2 = (\sqrt{x-5} + 1)^2$$

Square both sides.

$$x = x - 5 + 2\sqrt{x-5} + 1$$

Isolate the final radical.

$$4 = 2\sqrt{x-5}$$

$$4^2 = (2\sqrt{x-5})^2$$

Square both sides.

$$16 = 4(x-5)$$

Solve for x .

$$16 = 4x - 20$$

$$36 = 4x$$

$$x = 9$$

Note: See the third and fourth step above. $(\sqrt{x-5} + 1)^2 = (\sqrt{x-5} + 1)(\sqrt{x-5} + 1)$.

The FOIL process results in $2\sqrt{x-5}$.

- Know that a solution which does not satisfy the original equation is called an **extraneous solution**. (See Examples 11 and 13 on pages 53–54, and the following example.) Always check for extraneous solutions.

Example: $3x(x-1)^{1/2} + 2(x-1)^{3/2} = 0$

$$(x-1)^{1/2}(3x + 2(x-1)) = 0$$

$$(x-1)^{1/2}(5x-2) = 0$$

Factor out $(x-1)^{1/2}$.

Simplify.

$$x = 1 \text{ or } x = \frac{2}{5}$$

However, $x = \frac{2}{5}$ leads to taking the square root of a negative number when you plug it

back into the original equation. Therefore, the only solution is $x = 1$.

- Know how to solve absolute value equations. (See Example 14 on page 54.) Be sure to note that the absolute value equations you work with in this section will generally have two answers.

$$\text{Recall that } |a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Practice Problems

You will NOT turn these in.

Section 1.5

- Read page 44 through Example 1 on page 45.
 - Work page 55, exercises 1, 3, 9, 11, 15.
- Read from page 46, “Quadratic Equations,” through Example 6.
 - Work page 55, exercises 25, 27, 37, 43, 45, 47, 49.
- Read from page 49, the box about the Quadratic Formula, through Example 8.
 - Work page 55, exercises 59, 67, 69, 73.
- Read Example 9.
 - Work page 56, exercise 101.
- Read from page 52, “Other Equations,” through the paragraph following Example 11.
 - Work page 56, exercise 81.
- Read from page 53, the paragraph preceding Example 12, through Example 13.
 - Work page 56, exercises 87, 89, 91.
- Read Example 14.
 - Work page 56, exercises 95, 97.

Now you are ready to complete and turn in the graded assignment.

Graded Assignment



Please read the **Appendix** in the back of this study guide for essential Independent Study in Idaho policies and procedures, and forms you will need to successfully complete this course. You are responsible for understanding and following ISI policies and procedures. If there is anything on these pages you do not understand, contact the ISI office for clarification. Before starting the written assignment for Lesson 1, see the letter in your registration packet for your instructor's requirements: **how to format and submit lessons, number of lessons you may submit at one time, and lesson guidelines.**

Section 1.5, pages 55–56, exercises 4, 14, 44, 50, 70, 80, 82, 90, 96, 100.